

Hydrostatic Pressure on a Partially and Fully Submerged Vertical Rectangular Surface

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ABSTRACT. *Hydrostatic Pressure Systems allow for the measurement and development of hydrostatic force and center of pressure equations and values necessary to build and evaluate fluid systems. To demonstrate this capability, known masses were added to one end of an Edibon Hydrostatic Pressure System, and water was added into the system until the arm was level (balancing the moment about the pivot). The corresponding height of the water was recorded and used to calculate the center of pressure and hydrostatic force on the vertical rectangular quadrant. These values were plotted against the height of the water. This process was repeated for both partially and fully submerged surfaces. For the partially submerged surface, the center of pressure had a linear relationship with the height of the water, while there was a power relationship between these variables for a fully submerged surface. For the partially submerged surface, hydrostatic force had a power relationship with the height of the water, and, for a fully submerged surface, hydrostatic force had a linear relationship with the height of the water. Thus, in both cases, the data collected verified the relationships of center of pressure and hydrostatic force against height presented in the given equations. Because the moment about the pivot must be zero, the relationship between mass and height further confirmed these findings. In order to assess the accuracy of the measurements taken by the pressure system, the theoretical heights were compared to the experimental values of height. Because the slope of this relationship was nearly one, the accuracy of the pressure system was confirmed, and no calibration was needed. The plot of standard deviation between theoretical and measured height against measured height supports this result. **Keywords.** Hydrostatic Pressure System, hydrostatic force, center of pressure*

Hydrostatic pressure is the pressure exerted by a fluid at rest due to the weight of fluid above the surface. Hydrostatic pressure and its resulting force have a wide variety of applications. These applications are as widespread as assembly line processes to irrigation and studies of soil properties. Thus, it is important to be able to measure and develop equations for hydrostatic force due to pressure as well as the center of pressure at which this force acts (Çengal and Cimbala, 2014).

One device used to measure hydrostatic force is an Edibon Hydrostatics Pressure system. This device (fig. 1) is based on the principle that the sum of the moments about the pivot must be equal to zero. Thus, the moment due to the weight of the masses applied to the left end must be equal to the moment due to the hydrostatic force acting on the vertical rectangular quadrant (Humpherys, 1991). When known masses are applied to the end of the system, the pivot rotates. In order to balance the moment caused by the weight of the masses and return the pivot to equilibrium, water is added into the chamber of the pressure system.

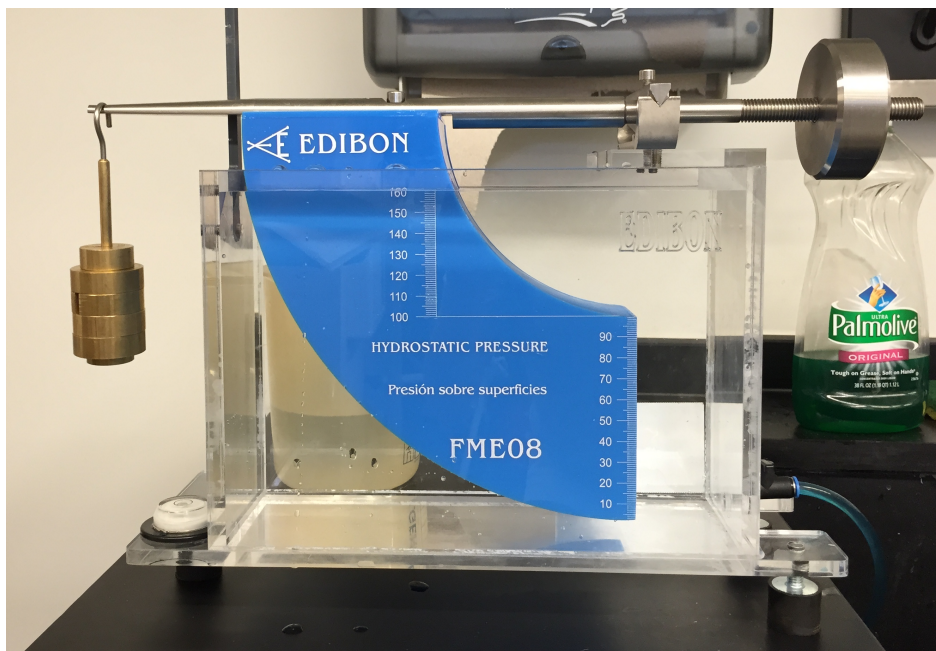


Figure 1. Edibon Hydrostatics Pressure Systems with weights applied to the arm (left), the pivot in equilibrium (right), and the chamber filled with water.

Using the known masses applied to the end of the system and the height of the water in the chamber, the center of pressure and hydrostatic force as well as mass can be determined using a series of equations. However, the relations for a partially submerged surface are different from those for a fully submerged surface. The relations for a partially submerged vertical rectangular surface are given in equations 1,2, and 3. The relations for a fully submerged rectangular surface are given in equations 4,5, and 6. For further clarification of the variables and values for the constants, figure 2 can be referenced.

$$m = \frac{\rho b}{2L} \left(a + d - \frac{h}{3} \right) h^2 \quad (1)$$

where

- m= mass applied to end of pressure system (kg)
- ρ = density of the fluid in the chamber (water) (kg/m^3)
- b= depth of surface where pressure is applied (m)
- L= length of balance bridge arm from weights to pivot (m)
- a= height from arm to top of lower surface (m)
- d= height of lower surface (m)
- h= height of fluid (water) in chamber (m)

$$h_p = a + d - \frac{h}{3} \quad (2)$$

where

- h_p = center of pressure measured from top (m)
- a= height from arm to top of lower surface (m)
- d= height of lower surface (m)
- h= height of fluid (water) in chamber (m)

$$F = \frac{1}{2} \rho g h^2 b \quad (3)$$

where

F= hydrostatic force acting on the vertical rectangular surface (N)
 ρ = density of the fluid in the chamber (water) (kg/m³)
g= acceleration due to gravity (m/s²)
h=height of fluid (water) in chamber (m)
b= depth of surface where pressure is applied (m)

$$m = \frac{\rho b d}{L} \left(a + \frac{d}{2} + \frac{d^2}{12(h-0.5d)} \right) (h - 0.5d) \quad (4)$$

where

m= mass applied to end of pressure system (kg)
 ρ = density of the fluid in the chamber (water) (kg/m³)
d= height of lower surface (m)
L= length of arm from weights to pivot (m)
a= height from arm to top of lower surface (m)
h= height of fluid (water) in chamber (m)

$$h_p = a + \frac{d}{2} + \frac{d^2}{12(h-\frac{d}{2})} \quad (5)$$

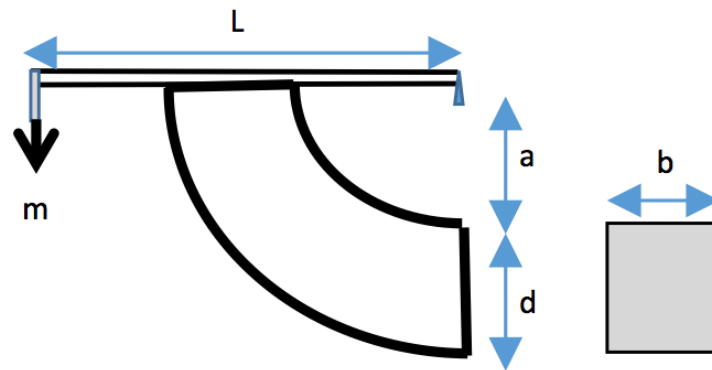
where

h_p = center of pressure measured from top (m)
a= height from balance bridge arm to top of lower surface (m)
d= height of lower surface (m)
h= height of fluid (water) in chamber (m)

$$F = \rho g b \left(h - \frac{d}{2} \right) d \quad (6)$$

where

F= hydrostatic force acting on the vertical rectangular surface (N)
 ρ = density of the fluid in the chamber (water) (kg/m³)
g= acceleration due to gravity (m/s²)
h=height of fluid (water) in chamber (m)
d= height of lower surface (m)



$a = 100 \text{ mm}$, $d = 100 \text{ mm}$, $b = 70 \text{ mm}$ and $L = 285 \text{ mm}$

Figure 2. Sketch of Edibon Hydrostatics Pressure System including variable clarification and values.

Once these values have been calculated, the relationships between height and mass, center of pressure, or hydrostatic force can be analyzed. Analysis of these plots is important in confirming that equations 1 through 6 properly represent the relationships between variables.

It is also important to analyze the quality of the data collected by the Edibon Hydrostatics Pressure System. Thus, using the known masses applied to the balance bridge arm, the theoretical height can be calculated using equations 1 and 4. Assuming these equations properly represent the system, this theoretical value of height can be compared to the height measured off of the pressure system. This relationship is representative of the quality of the data collected.

OBJECTIVES.

The objectives of this study are (a) to determine and analyze the relationships of hydrostatic force and center of pressure with respect to the height of the water in the pressure system chamber, (b) to confirm that these relationships are represented in the given equations, and (c) to assess the accuracy of the measurements produced by the Edibon Hydrostatics Pressure System.

METHODS AND MATERIALS.

DATA COLLECTION.

An Edibon Hydrostatics Pressure System was set on a flat surface. The chamber was filled with water until the balance bridge arm was level and the height read zero (at the bottom of the quadrant) (fig.3). Known masses (kg) were added to the left side of the arm. To balance the moment about the pivot, water was added into the chamber until the balance bridge arm was level again. The drain was also used to help level the arm. The corresponding water height (mm) was recorded.

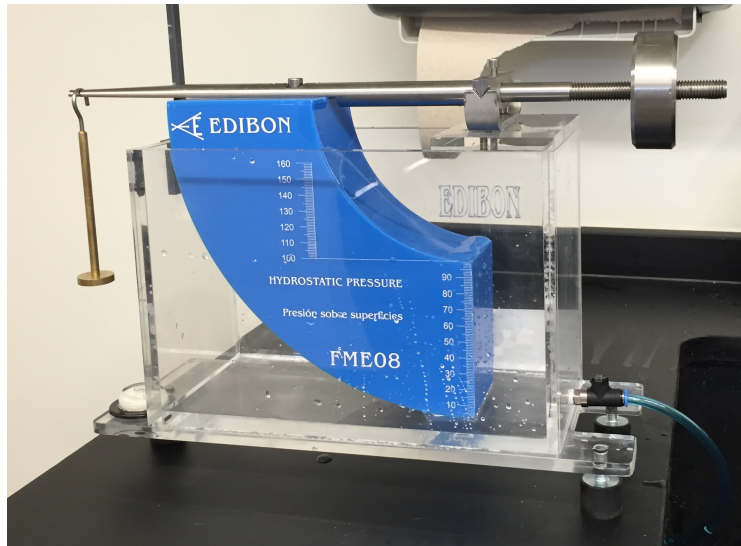


Figure 3. Edibon Hydrostatics Pressure System leveled at a water height of 0 mm.

This process was repeated for the partially submerged surface as mass was added in increments of 10 kg until the water height reached approximately 96 mm. At this point, water was added to that the vertical rectangular plate was fully submerged at 100 mm. The balance bridge arm was leveled at a water height of 100 mm by adding masses until the total mass reached approximately 225 g.

Now fully submerged, the known masses were added in increments of 50 g to the left side of the balance bridge arm. Water addition and the drain were used to level the balance bridge arm. The corresponding water height was recorded. This process was continued until the water height reached 160 mm which occurred with a mass of 450 g.

DATA ANALYSIS.

The water heights were converted into meters, and the masses were converted into kilograms using equations 7 and 8 respectively.

$$1 \text{ mm} = 0.001 \text{ m} \quad (7)$$

$$1 \text{ g} = 0.001 \text{ kg} \quad (8)$$

For the partially submerged surface (water heights under 100 mm), equations 1-3 were used to calculate theoretical height, the center of pressure, and the hydrostatic force acting on the surface respectively. The applied masses were then plotted against the corresponding theoretical heights, the theoretical heights were plotted against the measured heights for the same masses, the centers of pressure was plotted against the corresponding measured heights, and the hydrostatic forces were plotted against the corresponding measured heights.

The standard deviation was additionally calculated between the theoretical water heights and the measured water heights for each of the masses. A graph of the standard deviation against the measured height was developed for the partially submerged surface.

For the fully submerged surface (water heights greater than 100 mm), equations 4-6 were used to calculate theoretical height, the center of pressure, and the hydrostatic force acting on the surface respectively. Once again, these values were used to develop plots of various relationships (the applied masses versus the corresponding theoretical heights, the theoretical heights versus the measured heights for the same masses, the centers of pressure versus the corresponding measured heights, and the hydrostatic forces versus the corresponding measured heights).

The standard deviations between the theoretical heights and the measured heights of water for the fully submerged surface were calculated. The results were plotted against the corresponding measured heights.

RESULTS AND DISCUSSION.

Effect on Partially Submerged Surface.

In a plot of theoretical height versus measured height of water for a given mass (fig. 4), there is a linear relationship with a slope of approximately one for a partially submerged surface. This relationship fits the height measured from the Hydrostatic Pressure System with an R^2 value of 0.99982. Thus, the measured height is approximately equal to the theoretical height corresponding to the applied mass (the height measured by the pressure system is accurate). As a result, calibration of the pressure system is not needed.

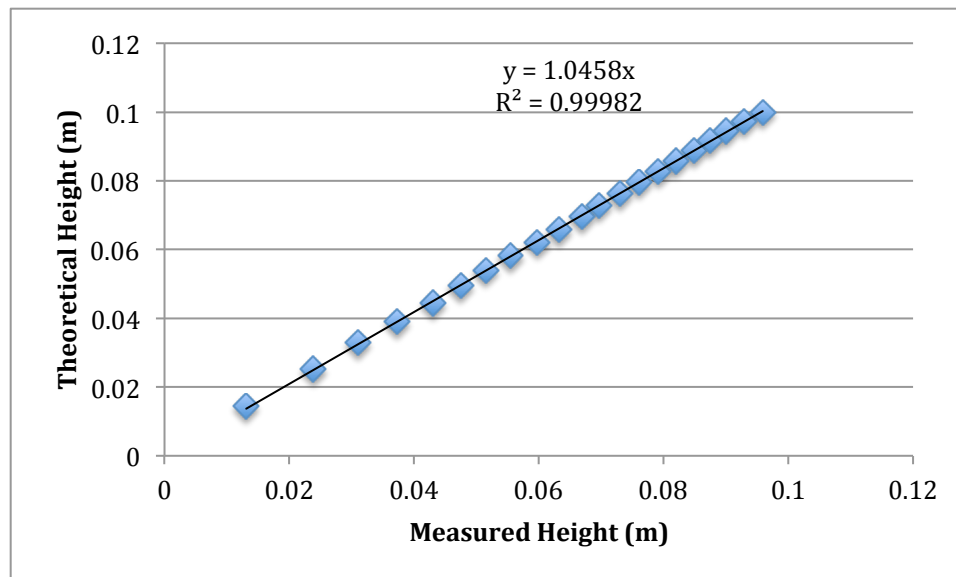


Figure 4. Theoretical Height versus Measured Height of water in chamber for a corresponding applied mass for a partially submerged surface.

The results given in figure 4 are further supported by the plot of the standard deviation between the theoretical and measured height of water versus the measured height of water in the chamber (fig. 5). The standard deviation varies little with change in water height (the plot is almost horizontal). Also, the measured height of the water is varied by less than 0.005 m from the theoretical value for height. Also, the percent error in height measurements remained below 7%: within the acceptable error of 15%. Thus, the heights measured from the pressure system are close in value to the theoretical heights. As a result, there is no need for calibration.

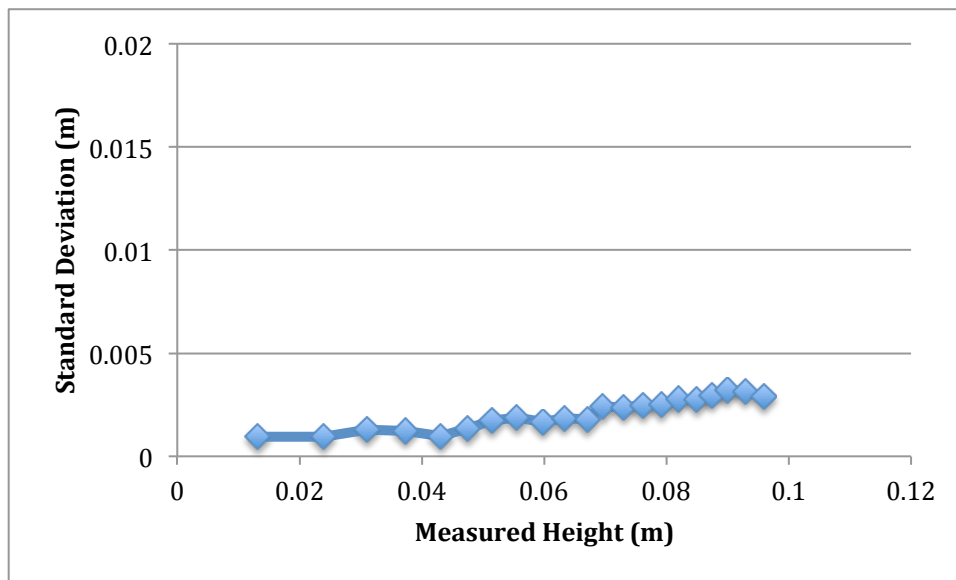


Figure 5. Standard Deviation between theoretical and measured height of water versus the measured height of water in the chamber for a partially submerged surface.

As demonstrated by figure 6, as the height of water in the chamber of the pressure system increases, the hydrostatic force on the vertical rectangular surface of the quadrant increases as well. A power model represents this relationship between hydrostatic force and measured height. . Due to the power relationship between variables, as height increases, each resulting change in height results in a larger increase in the value of hydrostatic pressure. The power model fits the data with an R^2 value of 1, so the model also supports the relation between hydrostatic force and height given in equation 3.

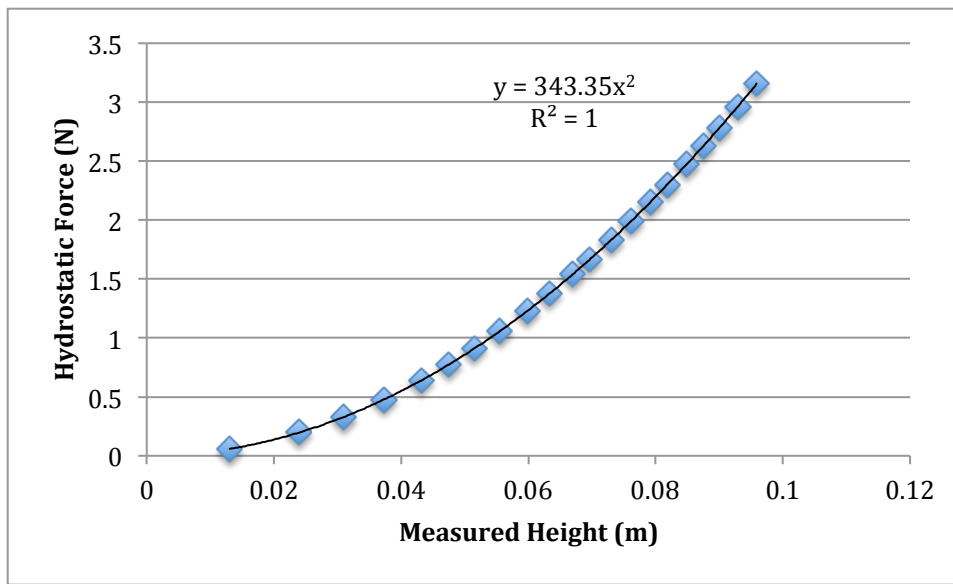


Figure 6. Relationship between hydrostatic force acting on the vertical rectangular surface and the measured height of water for a partially submerged surface.

Since the measured height is approximately equal to the theoretical height (fig. 4), a plot of the relationship between mass and theoretical height can be used for analysis as well. Since the moment about the pivot on the right end of the balance bridge arm must be equal to zero, the moment due to the weight of the applied masses is equal to the moment due to the hydrostatic force on the surface. As a result, as height increases, hydrostatic force increases (fig. 6), and mass (and thus weight) must increase as well (fig. 7). As a result, figure 7 is supportive of the results presented in figure 6. Figure 7 is also confirmation of the relationship between mass and height given in equation 1. Similar to equation 1, the model between mass and theoretical height is polynomial, which fits the collected data with an R^2 value of 0.99972.

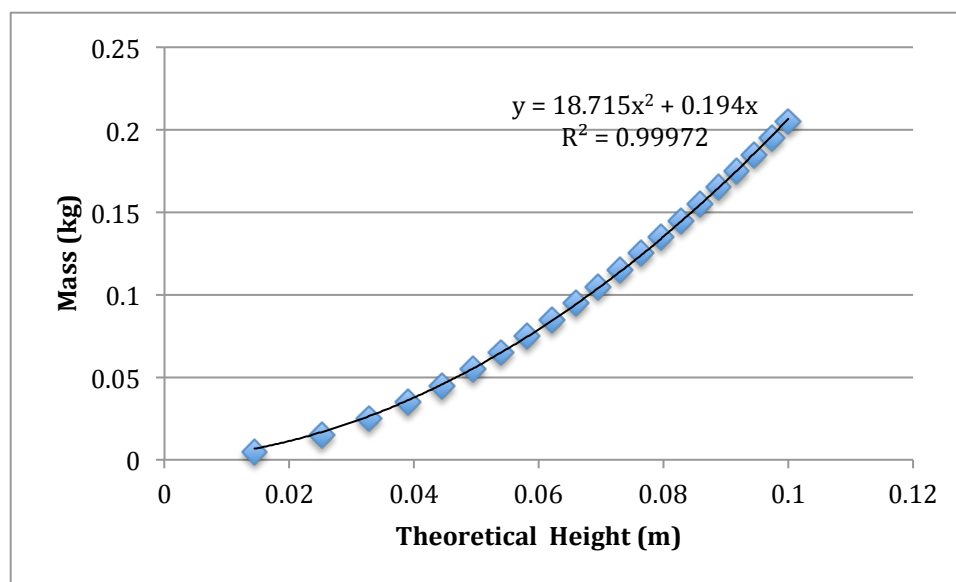


Figure 7. Relationship between applied mass and theoretical height of water in the chamber for a partially submerged surface.

The data collected from the hydrostatics pressure system is also able to provide information on the center of pressure on the vertical surface at various heights (fig. 8). Similar to the linear relationship between the center of pressure and height given in equation 2, the plot of the center of pressure versus the measured height of water for a partially submerged surface is a linear model supported by an R^2 value of 1. Thus, as water height increases in the chamber, the center of pressure (measured from the balance bridge arm down) decreases. In fact, as the height of the water in the chamber increases, the center of pressure approaches the centroid of the surface (0.15 m). Similar results were reported by Humpherys (1991) in his study of Center-of-Pressure Gates.

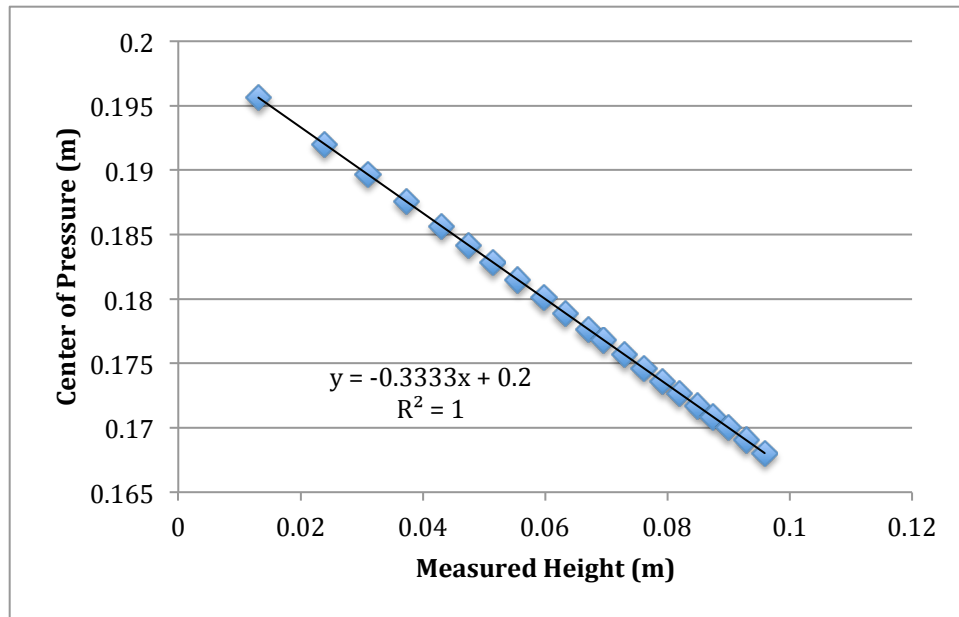


Figure 8. Center of pressure on the vertical rectangular quadrant versus the measured height of water in the chamber for a partially submerged surface.

The procedure used to determine the relationship of the center of pressure and hydrostatic force with height or depth of the water for a partially submerged surface is similar to a procedure used during study of sessile drops (Tadmor and Yadav, 2007).

Effect on Fully Submerged Surface.

Similar to the results found under the study of a partially submerged surface, the relationship between the theoretical and measured height for a fully submerged surface at a given applied weight is linear (fig. 9). This relationship fits the data gathered from the pressure system with an R^2 value of 0.9912. Since the slope of the plot is approximately one, the measured height of the water in the chamber is approximately equal to the theoretical value of the height of water at a given mass. Thus, no calibration is needed due to the relative accuracy of the measurements for a fully submerged surface.

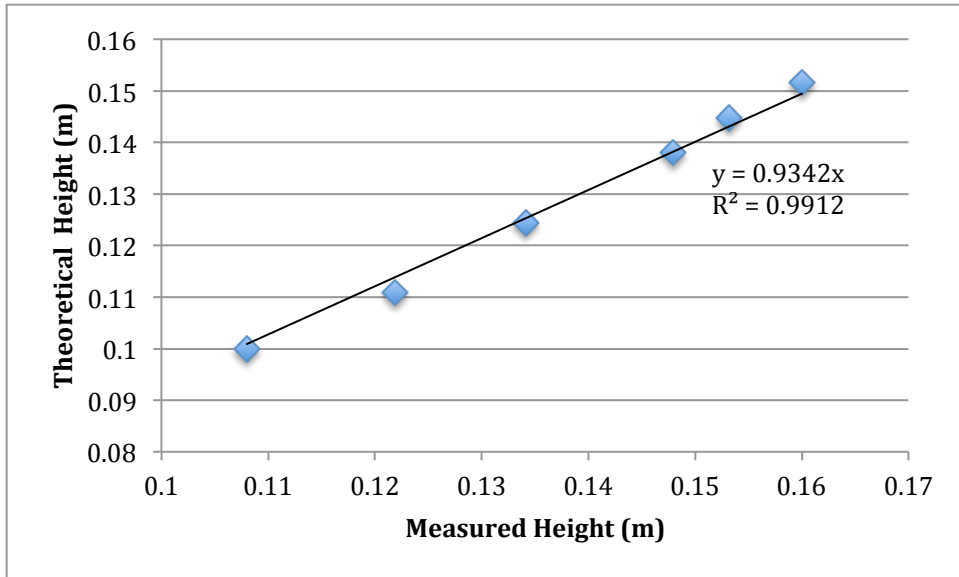


Figure 9. Theoretical height versus measured height of water in the chamber for a fully submerged surface.

The accuracy of the measurements taken by the hydrostatics pressure system is also supported by the plot of the standard deviation between theoretical height and measured height, and the measured height of water in the chamber. Like figure 5, the plot of standard deviation and measured height for a fully submerged surface is nearly horizontal (fig. 10). Because the standard deviation is less than 0.01 m at all heights, the measured height is relatively close in value to theoretical height. While the measured heights vary more from the theoretical heights for a fully submerged surface than the partially submerged surface, the variation is still low. In fact, the percent error in height measurements remained below 7%: well under the acceptable error of 15%. Thus, figure 10 is confirmation that the hydrostatics pressure system does not need to be calibrated.

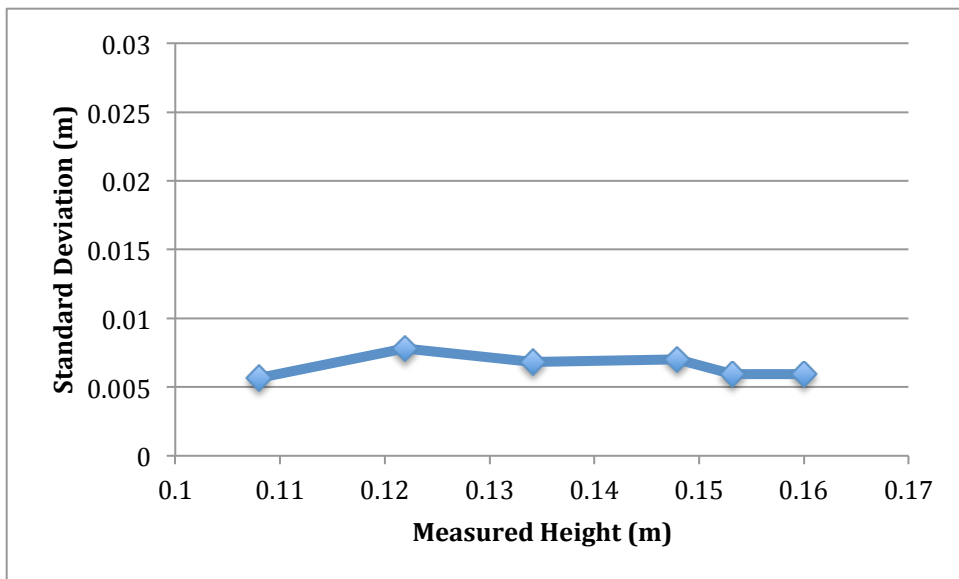


Figure 10. Standard Deviation between theoretical and measured height versus measured height of water in the chamber for a fully submerged surface.

Unlike the plot for hydrostatic force versus measured height of a partially submerged surface (fig. 6), the hydrostatic force versus measured height for a fully submerged vertical rectangular surface fits a linear model with an R^2 value of 1 (fig. 11). Thus, this model is also consistent with the relationship between height and hydrostatic force given in equation 6. As shown in the graph, hydrostatic force is increasing steadily with the increasing height of water in the chamber of the pressure system. Similar results were found in the study of slopes under drawdown conditions (Lane and Griffiths, 2000).

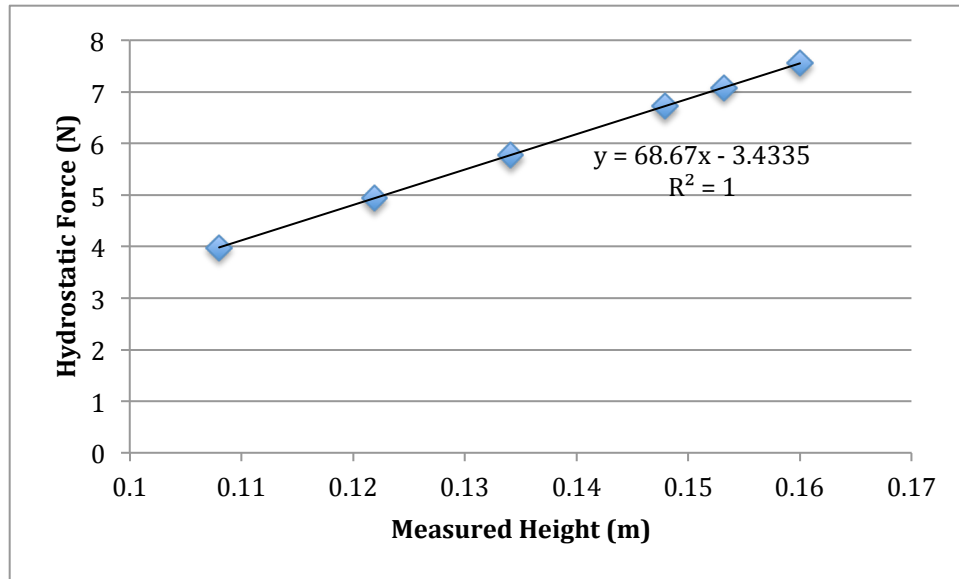


Figure 11. Hydrostatic force versus measured height of water in the chamber for a fully submerged surface.

Similar to figure 7, the plot of applied mass versus theoretical height for a fully submerged surface reaffirms the results of the plot for hydrostatic force versus measured height of the water (fig. 12). Since the theoretical height is approximately equal to the measured height, the plot is confirmation that, as the height of water in the chamber increases; the applied mass must also increase. This is once again due to the fact that the moments about the pivot must balance. As the height of water in the chamber increases, the applied mass must also increase to counteract the moment due to the hydrostatic force acting on the quadrant (fig. 11). However, unlike the plot for a partially submerged surface, the plot of applied mass versus theoretical height is linear for a fully submerged surface with an R^2 value of 0.99854. Thus, the relationship between mass and height given in equation 4 is supported by this plot.

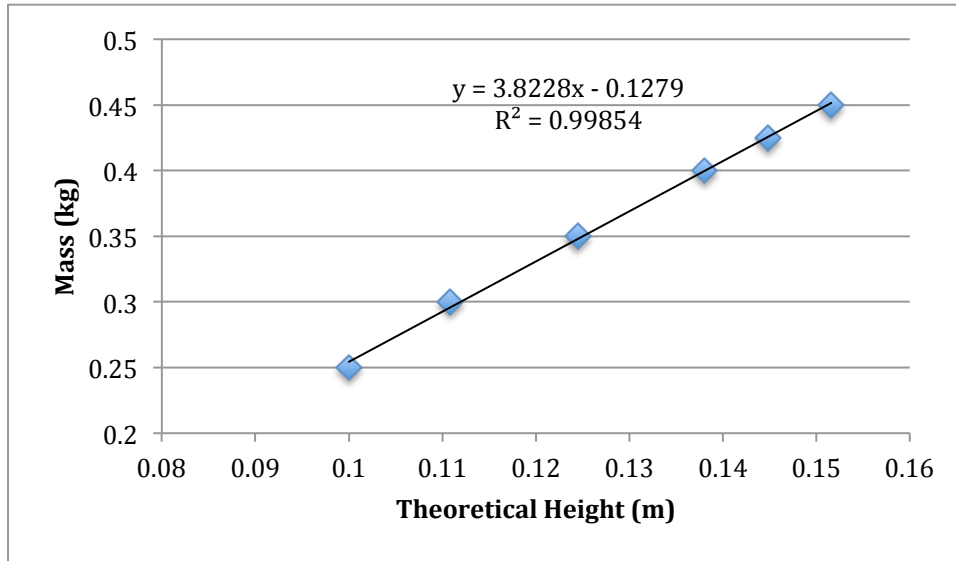


Figure 12. Relationship between applied mass and the theoretical height of water in the chamber for a fully submerged surface.

Unlike the graph of center of pressure versus measured height for a partially submerged surface (fig. 8), the plot of the center of pressure (measured from the balance bridge arm down) and measured height is a power model with an R^2 value of 0.98633 (fig. 13). Thus, this relationship is confirmation of the relationship between center of pressure and height given in equation 5. Similar to the partially submerged surface, as height increases, the center of pressure is decreasing towards to centroid of the vertical rectangular surface. In other words, the distance of the center of pressure from the arm decreases as the height of water in the chamber increases. These results are once again supported by the study of Center-of-Pressure gates (Humpherys, 1991).

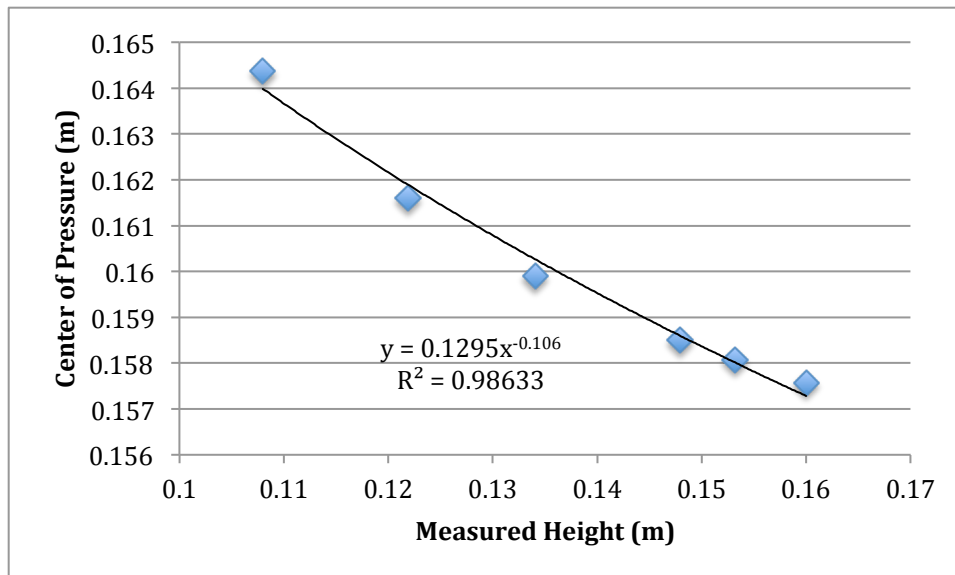


Figure 13. Center of pressure on the quadrant versus the measured height of water in the chamber for a fully submerged surface.

Causes for Error.

While in general the results seem to be in line with what is expected, there is still the possibility of error. This may be due to a variety of mistakes in the experiment. For example, there is the possibility of human error in reading when the balance bridge arm is level. This would lead to an inaccurate water height reading, which would consequently affect everything height was used to calculate. There may also have been human error in reading the height of the water in the chamber: also affecting the height measurement and all subsequent calculations.

Experimentally, a source of error may be in the possibility of water splashing onto the balance bridge arm while it was poured. This would cause an artificial increase in weight beyond the weight due to the applied masses. As a result, the hydrostatic force to counteract the masses moment would also be artificially high, and an artificially high water height would be read off the pressure system. Finally, the applied masses were not weighed prior to their application onto the balance bridge arm. Thus, the applied mass may weigh more due to accumulation of oils from being handled. As a result, an artificially high mass would be recorded, resulting in what appears to be a water height that is too high. However, these errors are so minor that it is likely that, even if they were present in the experiment, they would have little, to no, effect on the results.

CONCLUSIONS.

The Edibon Hydrostatics Pressure System accurately measures the height of the water in the chamber needed to calculate both hydrostatic force acting on the vertical rectangular quadrant and the center of pressure at which this force acts, with a low standard deviation from the theoretical water height for both partially and fully submerged surfaces. This is confirmed by the linear plots of theoretical versus measured water height in which the slope is approximately one for both the partially and fully submerged surfaces.

The data gathered from the pressure system also supports the relationships between variables as they are presented in the equations given to calculate hydrostatic force, center of pressure, and mass. In other words, the hydrostatic force acting on both partially and fully submerged vertical rectangular surface increases as the height of the fluid (water) in the chamber increases. This relationship is supported by the plots of mass versus theoretical height when the balance of the moments about the pivot is considered. For both partially and fully submerged surfaces, the center of pressure (measured from the balance bridge arm down) decreases towards the centroid of the quadrant as the height of water in the chamber increases.

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